

Background and Methodology in Simulation Analysis: OASim

Date:

November 27, 2023

Authors:

Tim Weaver, Josh Pyke

1 Introduction

OASim offers a robust set of tools that can be used to simulate many aspects of the organ allocation system (OAS) as it processes through a sequence of donors and candidate events. The framework of OASim may also be applicable to other systems that involve a queue along with rules for sorting the queue, but we investigate only matters related to organ allocation.

Given the robust nature of the software, a wide range of research questions can be investigated. Here we discuss possibilities and considerations when designing simulation studies.

2 Background: The Organ Allocation System

The OAS includes all aspects of the process of allocating donated organs to individuals who are waiting to receive a transplant. There are two main populations of people involved in this process: those waiting for an organ and those who have donated an organ (see Figure 1). In the figure, those who go on to wait for an organ are shown on the left track and those who donate an organ after death are shown on the right.

Starting from the general population, some individuals will develop disease that has the possibility of transplant as a treatment. Of these individuals, some will go on to receive treatment for the condition. As a part of their treatment, some may be referred for evaluation for transplant; of these individuals, some will ultimately be listed for transplant (that is, they will be added to an organ transplant waiting list). The population of individuals who have been listed and are waiting for a transplant are referred to as “candidates.”

Starting again from the general population, some individuals will be in a state of imminent death from disease or injury. Of these, some are at a hospital and able to be

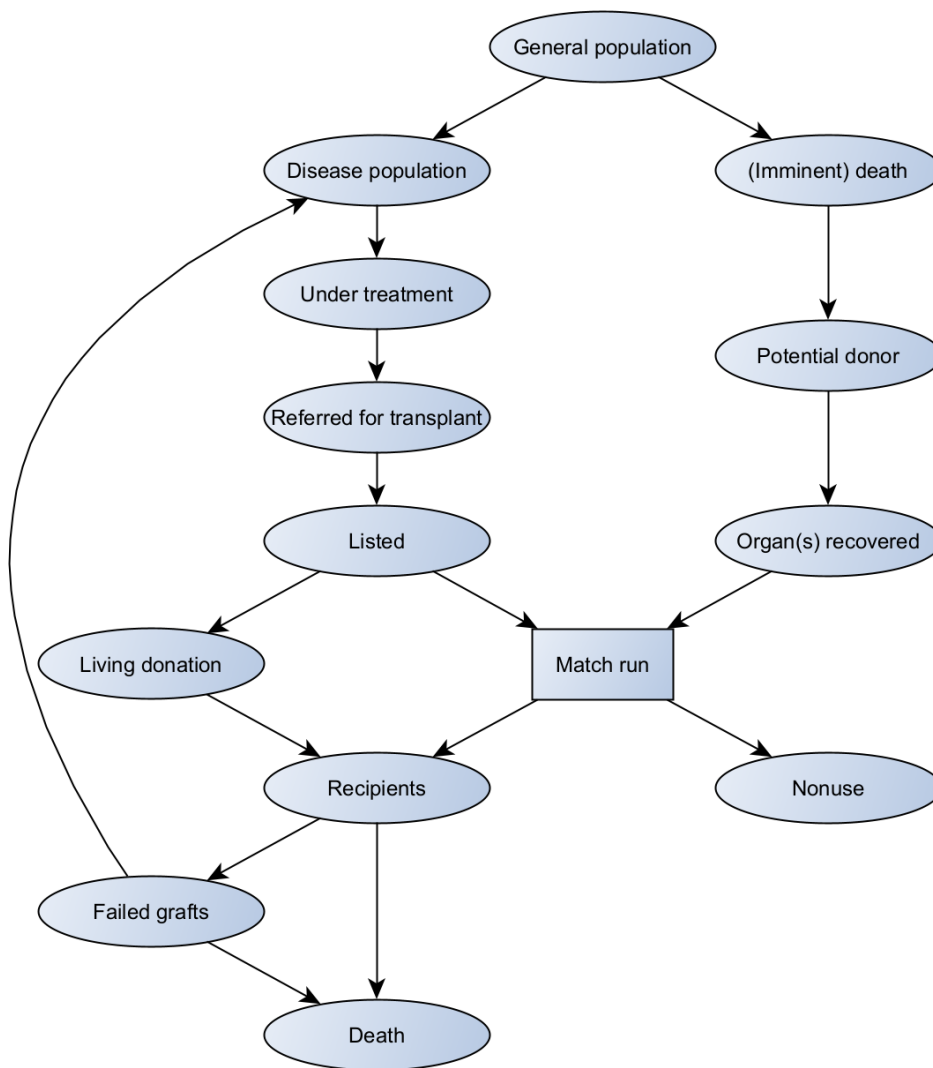


Figure 1: Entire Transplant Process

evaluated for potential donation of their organs. Of those individuals who have at least one organ that is a viable option for deceased donor transplant, some will have agreed to donation and go on to have the organ(s) recovered in preparation for allocation to an individual waiting for an organ (ie, a candidate).

At this point in the process, a match run (MR) can be performed; this is the sorting of a group of candidates into priority order for a given donated organ. The process sorts the candidates based on an allocation policy defined by Organ Procurement and Transplantation Network (OPTN) committees under the Final Rule. After the sorted results of the MR have been determined, the organ is offered to each candidate in order. The first candidate on the list (in concert with their treatment center) then has the option to accept the donated organ. If they choose to accept the organ, their transplant can proceed. If they decline the donated organ, it will be offered to the next candidate on the sorted MR list. If all candidates on the MR have declined the organ, it will not be used (nonuse)¹.

For some organs (eg, kidney), living donation is an additional pathway to transplant. In this process, candidates do not take part in MRs for donated organs but instead make arrangements for a living donor transplant.

Following transplant of either a deceased or living donor organ, individuals transition from being transplant candidates to "recipients," and their new organ is referred to as a "graft." This population may live with a functioning graft until death caused by injury or any other disease. These recipients may even have organ failure in other organs and repeat this same process for the additional organ. This same population may develop disease of their transplanted organ (failed graft). Those with graft failure have now transitioned back to the disease population at the top of the figure. Those who have rejoined the disease population may repeat the transplant candidate process as described above; in this case they are referred to as "re-listed candidates."

2.1 The Portion Modeled by OASim

The entire OAS is a broad system, and OASim focuses on a section of the overall processes (see Figure 2). The focus of OASim is on studying the *allocation* of donated organs with the MR process being the main step where this occurs. To this end, the simulation process starts at the point where candidates and recovered organs have already been identified; or, said another way, OASim takes as input a population of candidates and recovered organs.

¹Note: Declines by all candidates is not the sole reason for nonuse of a donated organ, but for simplicity it is the most relevant reason for the simulated environment.

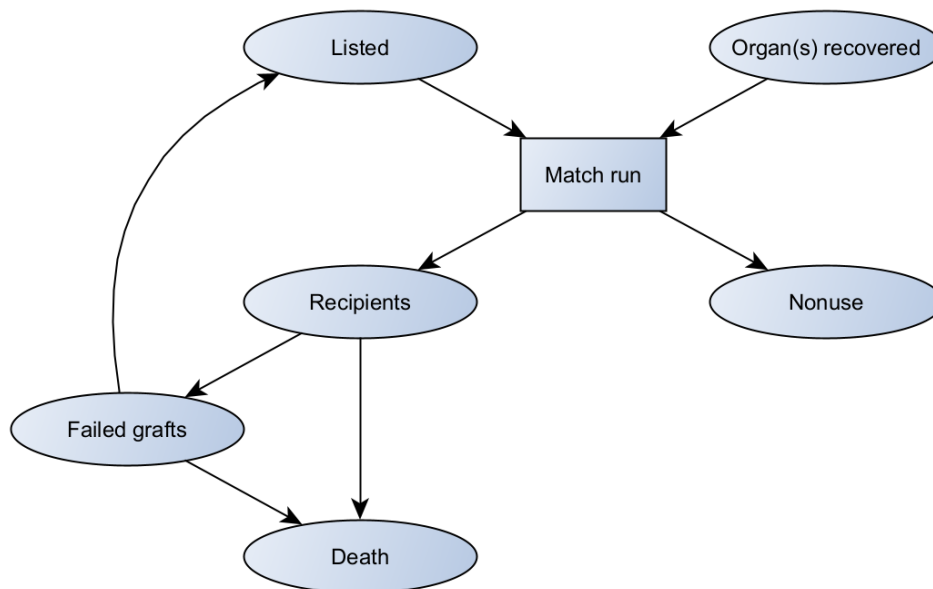


Figure 2: OASim Domain

3 Organ Allocation Simulation

3.1 What Is OASim?

This section of the document is based on sections 1-3 in the entry “Computer Simulations in Science” from the *Stanford Encyclopedia of Philosophy (Winter 2019 Edition)*. Here we summarize and describe the frameworks laid out in the encyclopedia entry specifically in terms of the OAS and OASim to answer this question: What is OASim?

3.1.1 A Narrow Definition

In a narrow sense, OASim is a computer program that uses step-by-step methods to explore the approximate behavior of the OAS (or more generally any system with a queue, sorting rules, and events that trigger the sorting). Given the state of the OAS at some initial time t , OASim uses a set of rules and instructions to calculate the state at $t+1$; from the state at $t+1$ it uses the rules to calculate the state at $t+2$, and so on. The step-by-step processing is a natural choice for the OAS because the system is largely recorded as a sequence of discrete events (eg, a candidate visits a clinic and has lab values updated, a candidate applies for and receives an exception, a donated organ arrives). The algorithm produces a numerical history of the evolution of the system’s state where the resultant “data” are meant to mimic a numerical history of the actual OAS.

From a user’s point of view, this would be a situation where they have installed the

OASim software and created all of their own input data and configuration files.

3.1.2 A Broad Definition

A broader definition of OASim may refer to the entire process of *an OASim study*; it is a comprehensive method for studying the OAS. In this framing, the narrow definition above is only a part of OASim, with all inputs and parameter settings that are processed by the computer program along with the presentation and study of the simulated data making up the rest of OASim. From a user's point of view, this would be a situation where they have installed the OASim software along with a set of input files that represent more aspects of the OAS (Figure 2). As an example, the population of candidates is represented by a dataset derived from historical records, and a statistical distribution is used to randomize when the candidates arrive.

This comprehensive method for studying the OAS may include:

- Choosing and accessing appropriate models to represent different components of the OAS (eg, statistical models of candidate acceptance decisions or input data randomization models)
- Implementing the components of the OAS as a computer program:
 - the statistical and data models associated with representing the OAS in a mathematical setting
 - parameters that represent the allocation rules of the OAS
 - instructions that control the computer program as it progresses through the sequence of events
- Running the "simulation" in the narrow sense of the definition above in order to create the simulated data
- Presenting and analyzing the resultant "data" to draw conclusions about the system

3.2 Types of Simulation in OASim

OASim allows for multiple types of simulation to be implemented. Here we discuss different simulation types and how they may be present in an OASim investigation.

Equation-Based

We do not believe that any components of OASim can be described as equation-based.

Agent-Based

In an agent-based simulation, each individual is modeled and has their own set of rules that govern their behavior. This is the only type of simulation that is guaranteed in every OASim study; donors and candidates are modeled at the individual (agent) level and have a set of rules that govern how they interact. Each candidate is represented as a dataset and each row in that dataset represents an event related to that candidate; as the simulation progresses, the population of candidates is updated one at a time for each candidate and each of their events. Similarly, each donor is represented as a row of a dataset and are processed one at a time; as the simulation processes a donor event, an MR is created and can be offered to candidates at the level of an individual candidate and individual donor.

Multiscale

Multiscale simulations combine models at different scales of description. An OASim simulation may incorporate models that operate at different scales. As has already been discussed, modeling is required at the individual candidate and donor levels of the system, but OASim allows for a broad range of calculation and there is potential for models to apply to groups of candidates—say, at the transplant-center level.

Monte Carlo

Monte Carlo simulations use randomness to calculate properties of a system, but the randomness itself is not under investigation. OASim offers a number of tools for stochasticity, and this may be an important feature of a simulation study design. For example, by randomizing the arrivals of the candidates in an OASim dataset, novel MRs can be created for a simulation. OASim also offers random number generators along with a rich expression syntax, so the options for introducing Monte Carlo techniques in a simulation study are very expansive.

3.2.1 Models in OASim

The types of simulation described above are implemented in OASim via models of the OAS (see Figure 3²). As mentioned, all OASim designs will involve some agent-based elements; models for randomized arrivals, as well as calculation of the MR, certainly happen at the individual agent level. Other models may be based on elements derived from the population level; the placement mechanism or history generation may be of this

²Note, this framing of models is not the only way the system could be described; it is representative of our understanding of most analyses of the OAS. This framing is shown to help describe the simulation system and OASim and represent our understanding of both, but it is not meant to say this is the only framing possible.

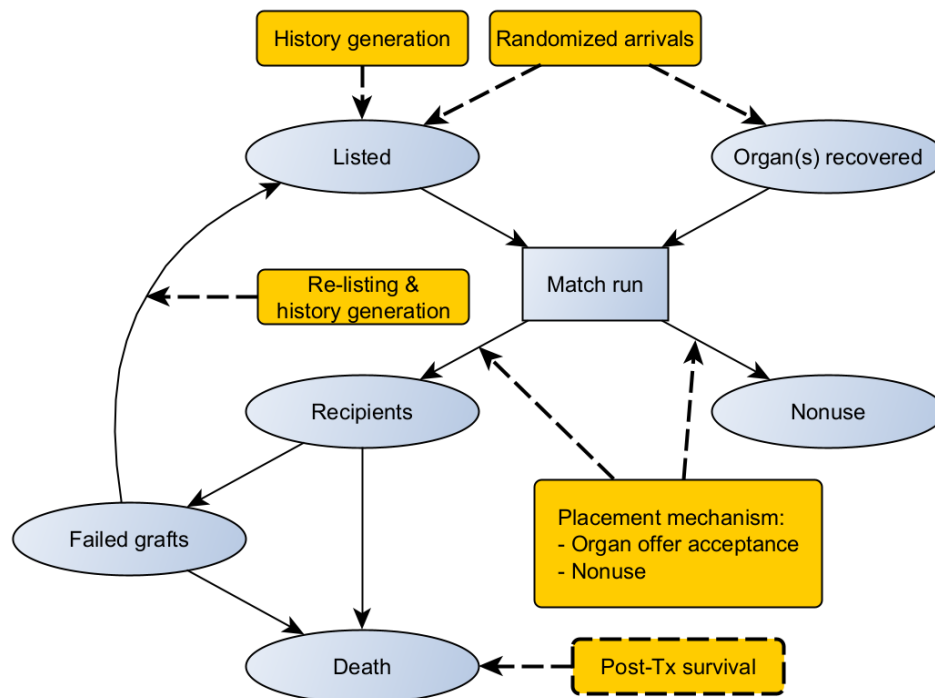


Figure 3: Annotated OASim Domain. Tx, transplant.

type. Stochasticity may be introduced into virtually any element of the OASim design; the randomized arrival model is a direct example of this type.

3.3 Why Simulate Organ Allocation

Heuristic Purposes

In this view of simulations, the point is to help understand the operation of the system, either for a broader public audience or for researchers within the transplant discipline.

Prediction

Here simulations are run in order to create data that we do not have access to. This is anticipated to be the main use for the OASim system. Simulating a change in allocation policy is a question of this type; the only data we have access to are historical records based on the existing allocation policy. Because of the nature of the policy, experimentation is not an option, so simulation may be used as a means to create scenarios within an OAS that cannot be observed in reality.

Understand the System and Its Behavior

Simulation studies of this type could be used to help understand how a current state of the OAS arrived, or what impacted the state. For example, if there is a variation in some

metric across the country, a range of simulations could be run to help understand which components would contribute to this outcome.

4 Types of Questions

The OASim framework is robust and allows a wide range of features of the OAS to be modeled and, in turn, allows for a wide range of questions to be investigated. In this section we discuss a number of questions that might be investigated using OASim. This is not meant to be an exhaustive list, and these techniques may, of course, be implemented together.

4.1 Past Simulation Studies of Organ Allocation

Historically, simulation studies of organ allocation undertaken by OPTN committees and the Scientific Registry of Transplant Recipients (SRTR) have used the simulated allocation models (SAMs) software. Here we will briefly describe features of historical simulation study designs, because they are well known to researchers in the field, are fairly constrained in scope and we anticipate future OASim studies will incorporate many of these features.

The main questions asked historically have related to changes in allocation policy. To address these questions, the studies have compared simulations run under different rules for organ allocation while keeping all other input data and settings the same between simulations. Data used to create models and run the simulation were drawn from SRTR, and the data available in the registry set boundaries on the domain of the simulation analysis. With this framework, the potential allocation rules are compared against the current allocation policy under a framework that is *meant to mimic a given historical period* as closely as possible. The simulations were backwards looking and created a predictive type of analysis. However, because the data conditions of the simulation were trained and tuned to historical data, the predictions are of a counterfactual nature and best described as predictions of what *would have* happened in the historical era under different (counterfactual) allocation policies.

An important component of the historical SAMs studies involved creating simulated data results across a range of potential historical possibilities. This was achieved by way of creating multiple input candidate and donor datasets sampled from historical data. The sampling was done to create randomized arrival times for both candidates and donors in order to create novel MRs that did not actually occur historically. This framework requires an assumption that the arrival of candidates and donors does not depend on the characteristics of the individuals; their arrivals are essentially random and so the reordering is thought to create valid counterfactual MRs. This randomization process was repeated a number of times to create a range of input datasets; these can be called

“iterations.” The simulation results using the range of input dataset iterations were then treated as if they were sampled from a larger distribution (of hypothetical potential MRs) and summary values were averaged across the iterations³.

This randomization of candidate and donor arrivals can be described as introducing a Monte Carlo aspect to the simulation, because the randomization of the arrivals is not under investigation in its own right. The stochasticity was only introduced in order to help calculate summary values (between different allocation policies) across hypothetical datasets.

Each allocation policy scenario under review for the study took the same randomized datasets described above as inputs. Another component of the SAMs models that introduced a stochastic element was the “acceptance model,” which was used to determine which (if any) candidate on the MR received a donated organ. This was implemented in such a way where an acceptance probability in (0, 1) was calculated based on a formula of a single candidate and donor characteristics based off of statistical modeling of the data cohort; a uniform variable was drawn to determine if the candidate “accepted” the organ. The sampling from this uniform distribution in this type of modeling of the OAS introduced an additional range in the simulated outcomes; given the same input data and settings, including a candidate and donor for an MR, different simulation scenario runs may have different outcomes for the “same” acceptance question⁴.

The current allocation policy, or the policy that was current during the timeframe of the data cohort, had a number of uses in this overall simulation framework. The first was as a “tuning” target for the component models within the broader OASim framework. Simulated runs of the historical time period were performed and certain outcome metrics compared against those calculated against the historical dataset, and modification of the acceptance model was used to bring the summary measures closer to those seen historically. The second was as a comparison group for the historical predictions, with the results often being limited in interpretation to directional changes only. That is, the simulated results of the alternative allocation policies were not compared to historical results. The simulated results of the different allocation policies were only compared between each other and the simulated results of the current allocation policy.

³Note: Even though the results were treated as if they were from a larger hypothetical distribution and the results averaged, very few assumptions of statistical distribution were made, and thus formal statistical testing was not undertaken.

⁴For a single scenario, multiple runs could be guaranteed to return the same results for a single MR by setting a random seed.

4.2 Varying Data Conditions

The previous section described the overall logic behind the questions asked and methods used to investigate features of the OAS in past simulations using the SAMs. The following sections describe additional questions that may be asked with the broader set of OASim tools.

We have described a study design that attempted to mimic as closely as possible a specific era in history in the simulation input dataset and compared across allocation policies⁵. In a design of this nature, prediction about the future would not be appropriate unless the future state of candidates and donors was assumed to not be changing in relation to the data cohort period. This assumption is often not valid. A study question related to future prediction would need to model some range of possibilities for the future state of the system. A study question interested in prediction could be phrased along these lines: "Given the current rules for allocation, *what is likely to happen* under potential future data conditions?"

Under a research question of this type, the range of simulated outcomes might come from varying the listing and donation trends of the candidates and donors: what happens if the listing trends do not change from today? what if the rate at which candidates list increases while the rate of donation decreases? and so forth. The details of creating the modeled (future) datasets would be the responsibility of the researcher; using the randomized arrivals framework from the last section with some sort of oversampling could be one method of achieving an input dataset that is appropriate for prediction.

4.3 Placement Mechanism Differences

Another component of the OAS that can be modeled within OASim is the mechanism of placing an organ with a candidate after an MR has occurred. The MR creates an ordered list of candidates, with each candidate having the possibility of accepting the organ. A stochastic process based on statistical modeling has already been applied, but a much simpler method would be to simply allocate the organ to the candidate who appeared first on the MR. A placement mechanism of this sort might not be best for broad inference or prediction but may be useful to a researcher to help establish a possible range of values; a simulation of the "first in list" placement might be used to access questions about who is prioritized under a given allocation policy.

Variations in placement mechanism might not be the main source of simulated vari-

⁵Note: Even though the input datasets were designed to mimic the historical data, they are still models and some elements of the datasets do not reflect every aspect of the historical data perfectly.

ability in a study but could aid in giving confidence that the simulation study (in the broad sense) is valid and the results can be used to make inferences about the real world.

Another example that has been discussed in historical simulation analysis has been related to travel distance after some allocation policy change. Under (simulated) policies that prioritize travel differently than historical policy, it is often wondered if acceptance decisions will change. This could be modeled as part of the range of simulated outcomes and become another iteration that the analysis can examine. A study with this design might use the historical acceptance model as a middle ground, with a model that highly values low travel distance as one extreme and another model that does not as the other extreme.

4.4 Simulate at the Extremes

The previous example illustrated what can be an important simulation technique: setting parameter values to extremes in order to simulate as wide a range of outcomes as possible. This approach will not be appropriate for all research questions but should be considered, in particular for situations where there is some a priori idea of what might happen (ie, acceptance decisions around travel might adjust after a policy change).

4.5 Any Component Model Can Be Varied

The above examples taken together show that any subcomponent of OASim that introduces a model also introduces the possibility for simulations across a range of the model's parameters.

5 Verification, Validation, and Credibility

This section outlines Sargent's "Verification and Validation of Simulation Models" paper in terms of OASim and the OAS. The entire text can be found [here](#). All quotations in this section come from this Sargent paper. A key point that will be repeated is that all verification and validation are only valid with respect to a given research question. In this section it will be important to make a distinction between OASim in the narrow sense (ie, restricting to a computer program) and the broad sense (ie, referring to an overall OASim study).

OASim studies will likely be used to aid in investigations on the OAS as well as for decision making, whether by researchers, SRTR, or OPTN committees. A key question will of course be whether the study and the results of the study are "correct."

Verification

"Ensuring that the computer program of the computerized model and its implementation are correct."

Under this definition, verification is a technical aspect of the overall process. In the narrow sense of OASim, verification is purely a software development task; it ensures that the software can correctly process a sequence of candidate and donor events. In the broader sense of an overall study, verification ensures that results from data modeling are correctly translated into instructions for OASim.

Validation

"Substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model."

An OASim study should be developed for a specific question (or type of questions), recall the "Types of Questions" section, and its validity determined with respect to that question. For example, an OASim study designed for prediction of (potential) future trends in the OAS would need validation related to future listing and donation trends; an OASim study designed to address counterfactual predictive questions would require validation related to comparison with historical records. Further, validity for one purpose does not (generally) imply validity for another. Validity needs to be determined within some acceptable range that is determined by the accuracy required of the study results to make inferences.

Credibility

"Developing in (potential) users the confidence they require in order to use a model and in the information derived from that model."

Careful documentation of all logical and analytic steps of verification and validation of an OASim study will be required to provide the information needed by users to evaluate the study for credibility.

5.1 Basic Approaches

Sargent outlines four decision-making approaches to simulation study validation.

1. The simulation development team determines validity
2. The user(s) are heavily involved with development team in deciding the validity
3. Independent verification and validation
 - Third party (independent) of both developers and users
 - Useful when simulation involves multiple teams
4. Scoring model: Sargent does not recommend this approach

In the context of OASim, the SRTR biostatisticians and software developers can be thought of as the simulation development team and the SRTR biostatisticians, independent researchers, and OPTN committees could be thought of as the users. There will likely not be a place for independent verification and validation that is undertaken by someone who would not be considered a researcher and thus fall under the “user” label.

5.2 Paradigm

Sargent’s paradigm for computer simulation verification and validation is shown in Figure 4.

The *problem entity* for an OASim study is the OAS under investigation. The problem entity may include historical records of organ allocation, or if the research question involves generating data⁶, may include counterfactual or future predictive situations. The *conceptual model* is the collection of all mathematical/logical/verbal representations of the OAS problem entity developed for a particular study. The conceptual model(s) could include models of where in a sorted MR an organ is allocated, models of historical candidate records, models of donor arrival trends, etc. The *computerized model* is the conceptual model implemented as a computer program. “The conceptual model is developed through an *analysis and modeling phase*, the computerized model is developed through

⁶Often a main reason why a simulation is undertaken.

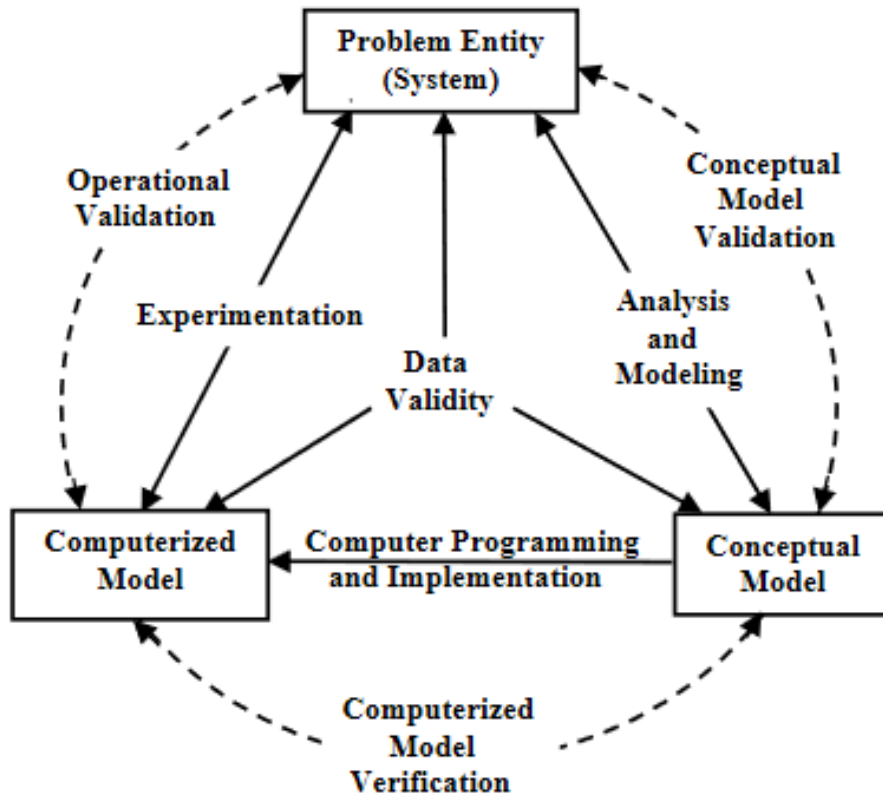


Figure 4: Simplified Version of the Modeling Process

a *computer programming and implementation phase*, and inferences about the problem entity are obtained by conducting computer experiments on the computerized model in the *experimentation phase*."

5.3 Conceptual Model Validation

The process of conceptual model validation is used to determine if⁷ "1) the theories and assumptions underlying the conceptual model are correct and 2) the model's representation of the problem entity and the model's structure, logic, and mathematical and causal relationships are 'reasonable' for the intended purpose of the model."

5.3.1 In the Narrow Sense

Recall that OASim operates as an agent-based simulation (ie, each candidate and donor is represented), where it processes a sequence of events. Within this framework is the assumption that the OAS under investigation can be represented as a sequence of discrete events; "time" is not important in and of itself, it is only used as a way to order the sequence of events. This basic assumption will be present in all OASim studies because it is built into the OASim software and needs to be considered when determining if an OASim study will be informative for a given research question.

5.3.2 In the Broad Sense

The conceptual model for an OASim study will (almost always) be made up of a number of submodels. Some examples that have been discussed include stochastic models of candidate (or donor) arrivals, history generation for transplanted candidates, and statistical models for placement of a donated organ to a position on the MR.

Examinations of the theories and assumptions underlying each model need to be performed using mathematical analysis and statistical methods with respect to data from the OAS under investigation. In the case of a statistical model for organ placement for example, statistical methods for model fit should be utilized that are appropriate for the model. This might include partitioning the data into training and test sets. Model assumptions such as independence of observations should be tested.

The result of this step in conceptual model validation will be a collection of analysis results that have validated each submodel on its own terms with respect to data from the OAS. Each of these analyses should essentially be stand-alone models of which validation

⁷Sargent uses "determine that"

can be interpreted using only data from the OAS and without making any reference to simulation or how the submodels will be used within the simulation study.

Following validation of each submodel in isolation, the research question of the OASim study is considered; each submodel along with their relationship to each other (ie, the overall model) are evaluated to determine if they are reasonable and correct for the specific research question. "This should include determining if the appropriate detail and aggregate relationships have been used for the model's intended purpose, and also if appropriate structure, logic, and mathematical and causal relationships have been used." For example, consider a study concerned with detailed changes around allocation policy in the short term compared to a study that was interested in predictions around long-term trends in waitlist size; the former might require a detailed history for each candidate, whereas in the latter study a simple model that only includes a single "listing" record per candidate might be appropriate. The submodels are then examined together to ensure that the precision required overall and by model can be achieved when the submodels are combined. Consider again the predictive study of long-term waitlist size; a detailed statistical organ placement model may be incompatible with the simple "listing" only candidate history model, so in this case a first-in-list placement mechanism may be sufficient for prediction of overall waitlist size. To further validate that the collection of submodels function together as expected, individual entities can be "traced" through the models. This involves examining how a candidate is recorded throughout the course of the simulation (not the realized values but the form the values would take).

5.4 Computerized Model Verification

The process of computerized model verification "is primarily concerned with determining that the simulation functions (e.g., the time-flow mechanism, pseudo random number generator, and random variate generators) and the computerized (simulation) model have been programmed and implemented correctly." OASim provides a special-purpose simulation language created using the higher level programming language C#. It was designed, developed, and implemented using modern software engineering techniques including object-oriented design, structured programming, and program modularity. The computerized model verification process is narrow in scope and focused on technical details related to implementation. The modular nature of OASim allows for the implementation of the distinct components of the OAS described in Figure 3 to be examined in isolation.

There are two basic approaches to computerized model verification: static testing and dynamic testing. Static testing involves analysis of the OASim input files without ac-

tually running the program. This may involve structured code reviews to avoid errors in implementation, comparisons between computerized implementation and the conceptual model representation to ensure the models have been translated correctly into a computer readable format, and examination of the structural relationships between the implemented submodels to ensure they accurately represent the intended relationships. Dynamic testing involves running the program under different conditions and examining calculated quantities to ensure they produce the expected results.

Comparisons in computerized model verification are quantitative in nature as there are predefined correct values for the results of the operations (eg, unit testing). Internal calculated values may also be examined during the program's run (ie, "debugging"). Comparisons between independent programming of the processes can also be used to ensure correct implementation. These methods may be applied at the level of individual calculation or aggregations may be used in cases of large numbers of comparisons. Finally, "[i]t is necessary to be aware while checking the correctness of the computer program and its implementation that errors found may be caused by the data, the conceptual model, the computer program, or the computer implementation."

5.5 Operational Validation

Operational validation involves running OASim with the submodels validated in the previous steps to determine if "the simulation model's output behavior has the accuracy required for the model's intended purpose over the domain of the model's intended applicability." Here the models are considered in tandem and so output behavior that does not behave as expected could be caused by any submodel or overall data quality issues. In this step the circular nature of the paradigm in Figure 4 comes into play. Deficiencies in output behavior should lead back to the submodels so remedies may be considered. If improvements can be made the process repeats. However, if on the other hand the submodels have been built as accurately as possible, the deficiencies in output behavior may be unavoidable; in this case, the discrepancies should be noted and used to put limits in interpretation of simulation inferences. As with the prior validation steps, the research question of interest needs to inform the operational validation. The metrics that will be used to make inferences about the problem entity should be used to examine the model behavior in the validation steps.

The data comparisons made during operational validation need to be carefully considered. If the OAS problem entity is observable, then direct comparisons between simulated and real-world output behavior can be made. Consider a study concerned with changes in allocation policy that is modeling a historical period (a what *would have hap-*

pened type of question); in this case a simulation(s) could be run to try and closely mimic the real-world historical data. Important metrics can then be compared between the generated and historical data using standard mathematical and statistical techniques.

However, the OAS problem entity is often not observable. Recall that simulation studies are often undertaken to generate data that is not available. It is anticipated that OASim studies will often be performed for this reason; the research question will be interested in prediction, either of the future or counterfactual situations from the past. In this case there is no real-world data available from the OAS that can be directly used for comparison. Exploring model output behavior across a range of input values can indicate if the OASim results are directionally correct as well as if the magnitudes of changes are reasonable. For example, with all other parameters held constant, would using an input dataset with a 10% higher donor arrival rate lead to more simulated transplants along with a smaller waitlist size?

5.6 Commentary

A key distinction in Figure 4 that should be emphasized is a clear separation between the conceptual model and the computerized model. The relationship between the two models is also important. Notice the arrow between the two models (computer programming and implementation) is the only one in the diagram that is uni-directional; in other words, the conceptual model directly defines the form of the computerized model, but the computerized model should not have influence on the conceptual model.

With any quantitative study it is easy to lose the distinction between the two models (conceptual and computerized) and the conceptual model ends up being contained “in the code”; in this case the problem entity (types of questions posed or even framing and vocabulary of discussions) can end up being influenced not by theory or analysis but by software implementation choices and details. This is an inappropriate direction for the inferences to take. This distinction is especially important when a simulation is being used to generate data that are unavailable.

6 References

- Winsberg, Eric, "Computer Simulations in Science", The Stanford Encyclopedia of Philosophy (Winter 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/win2019/entries/simulations-science/>. Accessed 11/27/2023.
- Sargent, Robert G., "Verification and Validation of Simulation Models", Proceedings of the 2011 Winter Simulation Conference, S. Jain, R.R. Creasey, J. Himmelspach, K.P. White, and M. Fu, (eds.), URL = <https://www.informs-sim.org/wsc11papers/016.pdf>. Accessed 11/27/2023.